

ESTIMATE OF TIME DEPENDENT PERFORMANCE ASSESSMENT INDEX OF GRADE POINT AVERAGES IN A SAMPLED POPULATION OF UNIVERSITY STUDENTS

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ABSTRACT

This paper proposes and developed a nonparametric statistical method for determining at what points in a series or sequence of trials or tests in time or space subjects are most likely to attain their highest (peak) or lowest (trough, in economic parlance) scores. Classification criteria and performance assessment index was also developed that would enable researchers, policy planners and implementers statistically gauge achievements by subjects and groups that could help informed introduction of necessary remedial intervention measures. A chi-square test statistic was developed to test any desired hypothesis. The proposed method is illustrated with some sample data and result showed that Chi-Square test statistic is statistically significance at 5% significant level thereby concluding that the difference between the proportions of undergraduate student of Electronics who on the average have highest and lowest scores through their four years of study in the University are not the same and hence are different. In conclusion, some undergraduate students of Electronics would therefore seem to need intensive and structured remedial measures to enable them enhance their academic performance in Electronics in the University.

KEYWORDS: chi-square test statistic, maximum scores, minimum scores, nonparametric statistics, performance.

INTRODUCTION

Invariably most people, groups or entrepreneurs sometimes during their life endeavor experience low levels of achievement often referred to as minimum points, a depression or trough in economics parlance, they also similarly experience high or optimum points also often in some situations referred to as booms or peaks in economics parlance (Blanche et al,2013;Cai et al,2011;Shen et al,2015).Therefore researchers, professional assessors, teachers, public health workers, strategic planners, economists, development planners and managers etc may often wish to determine at what point in time or space subjects exposed to a sequence of experiences, experiments, trials, tests or investments in a nation's economy attain their optimum or minimum performance levels or scores

depending on whether high or low scores are considered indices of success or failure for the trial or experiment of interest. In these situations, research interest may not be on the mean or the median scores or some measure of central tendency but rather on the lowest and highest scores in the sequence of trials, tests, experiments or investments. However lowest and highest values of some sets of data may not always exist or be unique. Furthermore, the lowest and highest values of a set of data unlike the mean and the median are often statistically intractable because of problems of uniqueness and of readily determining their distributions (*Blanche et al,2013; Hung and Chiang,2010a;2010b*). There are therefore not easily amenable for use with parametric statistical methods (*Hung and Chiang,2010a; Potapov et al,2012; Royston and Parmar,2011*). The problem of determining the highest and lowest values of a data set can of course be easily handled through the use of descriptive statistical methods (*Saha-Chaudhuri and Heagerty,2013; Song et al,2012; Lambert and Cherret,2014*). However, this approach will not enable statistical inferences and generalizations to be made (*Shen et al,2015; Saha and Heagerty,2010; Kalbfleisch and Prentice,2011*).

We here propose to develop a nonparametric statistical method for determining at what points in a series or sequence of trials or tests in time or space subjects are most likely to attain their highest (peak) or lowest (trough, in economic parlance) scores. Classification criteria and performance assessment index will also be developed that would enable researchers, policy planners and implementers statistically gauge achievements by subjects and groups that could help informed introduction of necessary remedial intervention measures. Test statistic will be developed to test any desired hypothesis.

The proposed method

Let $(x_{i1}, x_{i2}, x_{i3}, \dots, x_{iT})$ be the i th batch or block in a random sample of n observations drawn from T related population $X_1, X_2, X_3, \dots, X_T$ for $i = 1, 2, \dots, n$; where T is independent in time or space(location).Populations $X_1, X_2, X_3, \dots, X_T$ may be measurements on as low as the ordinal scale and need not be continuous or even numeric. The problem of research interest here is to determine at what points in time or space in a series of trials, experiments or tests subjects or groups are on the average likely to achieve their optimum(peak) and minimum(trough) levels of performance assessment indices and to determine the statistical significance of these indices. To do this statistically we may

Let

$$u_{it} = \begin{cases} 1, & \text{if } x_{it}, \text{ the highest score reported for the} \\ & \text{ith block is in treatment} \\ 0, & \text{if } x_{it}, \text{ is neither the highest nor the lowest} \\ & \text{scores recorded for the ith block is in treatment} \\ -1, & \text{if } x_{it}, \text{ the lowest score reported} \\ & \text{for the ith block is in treatment} \end{cases} \quad (1)$$

for $i = 1, 2, 3, \dots, n; t = 1, 2, \dots, T$

Note that $u_{it} = 0$ if all the scores in the i th block of subjects are the same for all treatment levels, location, time, period or population t , for $t=1, 2, \dots, T$.

Let

$$\pi_t^+ = P(u_{it} = 1); \pi_t^0 = P(u_{it} = 0); \pi_t^- = P(u_{it} = -1); \quad (2)$$

Where

$$\pi_t^+ + \pi_t^0 + \pi_t^- = 1 \quad (3)$$

Define

$$I_t = \sum_{i=1}^n u_{it} \quad (4)$$

And

$$I = \sum_{t=1}^T I_t = \sum_{t=1}^T \sum_{i=1}^n u_{it} \quad (5)$$

Now the expected value or mean and variance of u_{it} are respectively

$$E(u_{it}) = \pi_t^+ - \pi_t^-; \text{Var}(u_{it}) = \pi_t^+ + \pi_t^- - (\pi_t^+ - \pi_t^-)^2 \quad (6)$$

Also, the expected value or mean of I_t is

$$E(I_t) = \sum_{i=1}^n E(u_{it}) = n(\pi_t^+ - \pi_t^-) \quad (7)$$

And the variance of I_t is

$$\text{Var}(I_t) = \sum_{i=1}^n \text{Var}(u_{it}) = n(\pi_t^+ + \pi_t^- - (\pi_t^+ - \pi_t^-)^2) = n(\pi_t^+ + \pi_t^-) - \frac{I_t^2}{n} \quad (8)$$

Now π_t^+, π_t^0 and π_t^- are respectively the proportions of subjects or the probabilities that a randomly selected subjects earned his or her maximum, neither the maximum nor the minimum and the minimum score at treatment level location or time $t, t=1,2,\dots,T$. Their sample estimates are respectively

$$\hat{\pi}_t^+ = \frac{F_t^+}{n}; \hat{\pi}_t^0 = \frac{F_t^0}{n}; \hat{\pi}_t^- = \frac{F_t^-}{n} \quad (9)$$

where F_t^+, F_t^0 and F_t^- are respectively the number of 1's, 0's and -1's at the treatment level or time, t in the frequency distribution of the n values of these number in u_{it} for $i=1,2,\dots,n; t=1,2,\dots,T$. In other words

F_t^+, F_t^0 and F_t^- are respectively the number of times the maximum(highest), neither the maximum nor the minimum and the lowest (minimum) scores of all the subjects occurs in treatment level or time t such that

$$F_t^+ + F_t^0 + F_t^- = n \quad (10)$$

Now I_t is an index that provides a measure of how much the number of all the subjects whose maximum or highest (boom, peak)scores occur at treatment level or time t exceeds the number of all the subjects whose minimum or (lowest, trough, depression) scores occur at the same treatment level, location or time t, for some $t=1,2,\dots,T$. Positive values of I_t would indicate healthy performance or progress while negative values would indicate poor performance or retrogression at time 't'. The higher or larger and positive I_t is the higher the peak or boom in economic parlance the lower or smaller and negative. It is the deeper is the trough or economic depression at location or time t, for some $t=1,2,\dots,T$. Zero values of I_t would indicate stagnation at time 't' for some $t=1,2,\dots,T$. Now $\pi_t^+ - \pi_t^-$ is the difference between the proportion of subject or between the probabilities that a randomly selected subject earns his or her highest or maximum(peak, boom) score at treatment level location or time t and the probability that the same randomly selected subject earns his or her minimum lowest(trough, economic depression) score at the same treatment level, location or time t for some $t=1,2,3,\dots,T$. Its sample estimate is from Equations 7 and 9

$$\hat{\pi}_t^+ - \hat{\pi}_t^- = \frac{I_t}{n} = \frac{F_t^+ - F_t^-}{n} \quad (11)$$

where

$$I_t = F_t^+ - F_t^- = n(\hat{\pi}_t^+ - \hat{\pi}_t^-) \quad (12)$$

The sample estimate of the variance of $\hat{\pi}_t^+ - \hat{\pi}_t^- = \frac{I_t}{n}$ is from equation (8)

$$Var(\hat{\pi}_t^+ - \hat{\pi}_t^-) = Var\left(\frac{I_t}{n}\right) = \hat{\pi}_t^+ + \hat{\pi}_t^- - \frac{(\hat{\pi}_t^+ - \hat{\pi}_t^-)^2}{n} \quad (13)$$

Research interest may often be to determine whether subjects or group of subjects perform better than, worse at treatment level or time t, that is whether the index of performance I_t indicate a performance progression at treatment location or time 't'. In other words, the null hypothesis of interest here may be

$$H_0 : \hat{\pi}_t^+ - \hat{\pi}_t^- \geq \theta_{to} \text{ versus } H_1 : \hat{\pi}_t^+ - \hat{\pi}_t^- < \theta_{to}, \text{ say } (\theta \leq \theta_{to} \leq 1) \quad (14)$$

For some $t=1,2,\dots,T$. If the null hypothesis of equation 14 is true then the test statistic

$$\chi_t^2 = \frac{(\hat{I}_t - n.\theta_{to})^2}{Var(\hat{I}_t)} = \frac{n((\hat{\pi}_t^+ - \hat{\pi}_t^-) - \theta_{to})^2}{\hat{\pi}_t^+ + \hat{\pi}_t^- - (\hat{\pi}_t^+ - \hat{\pi}_t^-)^2} \quad (15)$$

Under the null hypothesis H0 of equation (14) has approximately the Chi-square distribution with 1 degree of freedom for sufficiently large sample size n and may be used to test this null hypothesis, for t=1,2,...,T. The null hypothesis H0 of equation (14) is rejected at the α level of significance if

$$\chi_t^2 \geq \chi_{1-\alpha;1}^2 \quad (16)$$

Otherwise H0 is accepted. However perhaps of more general research interest would be to determine whether subjects or groups of subjects on the average perform better than worse for all treatment location or overall for the study time periods or that performance indices are at least positive for all time period. That is, a desired null hypothesis may be that the highest score is as likely to be greater than the lowest score in any one treatment level or time period as in another. Thus a more general null hypothesis may be

$$H_0 : \pi_1^+ - \pi_1^- = \pi_2^+ - \pi_2^- = \dots = \pi_T^+ - \pi_T^- = \pi^+ - \pi^- \geq \theta_0$$

Versus

$$H_1 : \pi_t^+ - \pi_t^- < \theta_0, (\theta \leq \theta_0 \leq 1) \quad (17)$$

for some t = 1, 2, ..., T.

Where π^+, π^0 and π^- are respectively the common probabilities in the sampled populations that subjects or groups of subjects on the average earn the highest, neither the highest nor the lowest and the lowest scores in any treatment level or time period of the study. To develop a test statistic for the null hypothesis H0 of Equation (17) we note from Equation 5 that the expected value or mean of index I is

$$E(I) = \sum_{t=1}^T E(I_t) = n \sum_{t=1}^T (\pi_t^+ - \pi_t^-) \quad (18)$$

And the variance of I is

$$Var(I) = \sum_{t=1}^T Var(I_t) = n \sum_{t=1}^T (\pi_t^+ + \pi_t^- - (\pi_t^+ - \pi_t^-)^2) \quad (19)$$

Now under the null hypothesis of equation (17) we would expect the highest and lowest scores to be as likely to occur in any one treatment level or time period as in another. Hence under H0 we would have from Equations 18 and 19 that

$$E(I) = n.T(\pi^+ - \pi^-) \quad (20)$$

And

$$Var(I) = n.T(\pi^+ + \pi^- - (\pi^+ - \pi^-)^2) \quad (21)$$

Whose sample estimate under the null hypothesis H0 of Equation 17 are respectively

$$\hat{I} = n.T(\hat{\pi}^+ - \hat{\pi}^-) \quad (22)$$

And

$$Var(\hat{I}_t) = n.T(\hat{\pi}^+ + \hat{\pi}^- - (\hat{\pi}^+ - \hat{\pi}^-)^2) = n.T(\hat{\pi}^+ + \hat{\pi}^-) - \frac{\hat{I}^2}{n.T} \quad (23)$$

Where the common sample properties $\hat{\pi}^+$, $\hat{\pi}^0$ and $\hat{\pi}^-$ are respectively

$$\hat{\pi}^+ = \sum_{t=1}^T \frac{\hat{\pi}_t^+}{T} = \sum_t \frac{F_t^+}{n.T} = \frac{F^+}{n.T}; \hat{\pi}^0 = \sum_{t=1}^T \frac{\hat{\pi}_t^0}{T} = \sum_{t=1}^T \frac{F_t^0}{n.T} = \frac{F^0}{n.T}; \hat{\pi}^- = \sum_{t=1}^T \frac{\hat{\pi}_t^-}{T} = \sum_t \frac{F_t^-}{n.T} = \frac{F^-}{n.T} \quad (24)$$

Note that under the null hypothesis H0 of Equation 17, the sample estimate of $\pi^+ - \pi^-$, the difference between the common population proportions of maximum and minimum scores of all treatment levels locations or time periods is Equation (18)

$$\hat{\pi}^+ - \hat{\pi}^- = \frac{\hat{I}}{n.T} = \frac{F^+ - F^-}{n.T} \quad (25)$$

Whose sample variance under H0 is from equation (19) is

$$Var(\hat{\pi}^+ - \hat{\pi}^-) = \frac{Var(\hat{I})}{n^2.T^2} = \frac{\hat{\pi}^+ + \hat{\pi}^- - (\hat{\pi}^+ - \hat{\pi}^-)^2}{n.T} \quad (26)$$

Now under the null hypothesis H0 of Equation 17 the test statistic

$$\chi^2 = \frac{\left(\left(\hat{\pi}^+ - \hat{\pi}^-\right) - \theta_0\right)^2}{\text{Var}\left(\hat{\pi}^+ - \hat{\pi}^-\right)} = \frac{\left(\hat{I} - n.T\theta_0\right)^2}{\text{Var}\left(\hat{I}\right)} = \frac{n.T\left(\left(\hat{\pi}^+ - \hat{\pi}^-\right) - \theta_0\right)^2}{\hat{\pi}^+ + \hat{\pi}^- - \left(\hat{\pi}^+ - \hat{\pi}^-\right)^2} \quad (27)$$

Under the null hypothesis H0 of Equation 17 has approximately the Chi-Square distribution with T-1 degrees of freedom for sufficiently large sample size n and may be used to test the null hypothesis. The null hypothesis H0 is rejected at the α level of significance if

$$\chi^2 \geq \chi_{1-\alpha; T-1}^2 \quad (28)$$

Otherwise H0 is accepted.

In general, the null hypothesis of Equation 17 is tested. Its rejection would indicate a need for further analysis and perhaps the testing of the null hypothesis of Equation (14) to discover which treatment level or treatment levels may have led to the rejection of the more general null hypothesis of Equation (17). Note that although the null hypothesis of Equation (14) may be tested using a critical Chi-Square value with 1 degree of freedom, it is however recommended that to avoid accepting a false null hypothesis and committing Type II Error too frequently the calculated Chi-Square value be compared with a critical Chi-Square value with T-1 degrees of freedom.

ILLUSTRATIVE EXAMPLE

We use data on the grade point averages (GPAS) of a random sample of 24 Undergraduate students during each of their four years of studies for an Undergraduate Degree Electronics in a University (Table 1).

Table 1 (PA'S of a random sample of Undergraduate students for an Undergraduate degree in Electronics by year in a University.)

S/N	YEAR 1	YEAR 2	YEAR 3	YEAR 4
1	4.2	3.1	2.5	3.8
2	3.7	3.3	4.3	4.3
3	3.7	2.9	4.1	3.6
4	2.8	2.1	3.1	3.3
5	3.7	2.9	2.8	4.0
6	4.1	2.7	4.0	3.9
7	3.0	2.8	2.6	4.0
8	3.5	2.5	3.7	3.7

9	3.5	3.1	4.0	3.9
10	4.5	4.4	4.6	4.7
11	4.0	3.4	4.3	4.2
12	3.8	3.5	3.9	4.0
13	3.4	3.0	4.0	4.6
14	3.9	4.0	4.4	4.7
15	4.0	3.4	3.7	4.3
16	3.4	2.8	3.6	4.0
17	3.3	2.6	3.4	4.0
18	3.3	2.4	3.1	3.6
19	4.3	4.1	4.4	4.1
20	3.8	2.8	4.3	4.9
21	3.3	3.1	3.6	3.6
22	4.1	4.0	4.4	4.3
23	3.8	3.3	4.4	4.6
24	3.7	1.7	2.2	4.0

To illustrate the proposed method we apply Equation 1 to the GPAs in Table 1 to obtain values of u_{it} , for $i=1, 2, \dots, 24; t=1, 2, \dots, 4$ as shown in Table 2.

Table 2 (Values of u_{it} (Equation 1) for student's GPAs in Table (1).)

S/N	u_{i1}	u_{i2}	u_{i3}	u_{i4}	Highest score at:	Lowest score at:	Time gap (time unit)	Classification (direction)
	Year 1	Year 2	Year 3	Year 4				
1	1	0	-1	0	t_1	t_3	2	Worse (negative)
2	0	-1	1	1	t_3, t_4	t_2	1	Improved (positive)

3	0	-1	1	0	t_3	t_2	1	*
4	0	-1	0	1	t_4	t_2	2	Improved(positive)
5	0	0	-1	1	t_4	t_3	1	Improved(positive)
6	1	-1	0	0	t_1	t_2	-1	Improved(positive)
7	0	0	-1	1	t_4	t_3	1	Improved(positive)
8	0	-1	1	1	t_3, t_4	t_2	1	Improved(positive)
9	0	-1	1	0	t_3	t_2	1	*
10	0	-1	0	1	t_4	t_2	2	Improved(positive)
11	0	-1	1	0	t_3	t_2	1	*
12	0	-1	0	1	t_4	t_2	2	Improved(positive)
13	0	-1	0	1	t_4	t_2	2	Improved(positive)
14	-1	0	0	1	t_4	t_1	3	Improved(positive)
15	0	-1	0	1	t_4	t_2	2	Improved(positive)
16	0	-1	0	1	t_4	t_2	2	Improved(positive)
17	0	-1	0	1	t_4	t_2	2	Improved(positive)
18	0	-1	0	1	t_4	t_2	2	Improved(positive)
19	0	-1	1	-1	t_3	t_2, t_4	-1	worse(negative)
20	0	-1	0	1	t_4	t_2	2	Improved(positive)
21	0	-1	1	1	t_3, t_4	t_2	1	Improved(positive)
22	0	-1	1	0	t_3	t_2	1	*
23	0	-1	0	1	t_4	t_2	2	Improved(positive)

24	0	-1	0	1	t_4	t_2	2	Improved(positive)
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Summary values of u_{it} (Equation 1) of Table (2) are presented in Table (3).

Table 3 (Summary values of u_{it} (Equation 1) of Table (2).)

	u_{i1}	u_{i2}	u_{i3}	u_{i4}	F
F_t^+	2	0	8	17	27(F^+)
F_t^0	21	4	13	6	44(F^0)
F_t^-	1	20	3	1	25(F^-)
n	24	24	24	24	96(=n.T)
$\hat{\pi}_t^+$	0.083	0.00	0.333	0.708	0.281($\hat{\pi}^+$)
$\hat{\pi}_t^0$	0.875	0.167	0.542	0.250	0.458($\hat{\pi}^0$)
$\hat{\pi}_t^-$	0.042	0.833	0.125	0.042	0.260($\hat{\pi}^-$)
\hat{I}_t	1	-20	5	16	2(\hat{I})
$Var(\hat{I}_t)$	2.952	3.336	9.96	7.344	51.898(=var(\hat{I}))
χ_t^2	8.469	58.753	0.100	13.617	9.326

Table (3) shows the values of $\hat{\pi}_t^+, \hat{\pi}_t^0, \hat{\pi}_t^-, I$ and other statistics. The expected performance assessment index $\hat{I}_t = n(\hat{\pi}_t^+ - \hat{\pi}_t^-)$ is said to be at its lowest and negative in year two with $\hat{I}_2 = -20$, and rising slowly to its highest and positive value in year fourth with $\hat{I}_4 = 16$. Thus, the undergraduate students of Electronics on the average perform the poorest in year two while the students average performance then rises slowly to its highest level in year four. These results seem to indicate that under graduate students of Electronics understudy may on the average need some appropriate remedial actions especially before their penultimate years in the University. Now using the values of Table (3.) Equation 27 and assuming $\hat{\theta}_0 = 0.25$, the overall Chi-Square test statistic for the null hypothesis H_0 of Equation (17) is calculated using Equation 27 as (see Table 3) $\chi^2 = 9.326$ ($p - value = 0.3452$) which with $4-1 = 3$ degrees of freedom is statistically significant at the 5 % significant level ($\chi_{0.95;3}^2 = 7.815$).

We therefore reject the null hypothesis H_0 of Equation (17) and conclude that the difference between the proportions of undergraduate student of Electronics who on the average have highest and lowest scores through their four years of study in the University are not the same and hence are different. Additional tests of the null hypothesis H_0 of Equation (14) with $\theta_{t_0} = 0.25$, for $t = 1, 2, 3, 4$ used the Chi-Square values also shown in Table 3 for each of the four years of study. The calculated Chi-Square values when compared with the tabulated Chi-Square with 3 degrees' freedom ($\chi_{0.95;3}^2 = 7.815$) indicates that differential performances in years 1, 2, and 4 are statistically significant at 5% significance level and may be responsible for the significance of the overall Chi-Square value and hence for the observed differences in performance over the years and hence responsible for the rejection of the initial or overall null hypothesis H_0 of Equation (17). Only the differential performance by students in year 3 is found not to be statistically significant and hence may not be statistically different from 0.25. Finally note from the classification of students by performance also shown in Table 2, that most of the students improved their scores after their second year of study with one student progressively earning higher scores over all the years. In fact, only three students (shown with ***) got worse in their performance over the years and four students (shown with *) seemed to be inconsistent in their performance having performed poorly in year four. These seven undergraduate students of Electronics would therefore seem to need intensive and structured remedial measures to enable them enhance their academic performance in Electronics in the University.

SUMMARY AND CONCLUSION

We have in this paper proposed to develop a nonparametric statistical method for determining at what points in a series or sequence of trials or tests in time or space subjects are most likely to attain their highest (peak) or lowest (trough, in economic parlance) scores. Classification criteria and performance assessment index was developed to enable researchers, policy planners and implementers statistically gauge achievements by subjects and groups that could help informed introduction of necessary remedial intervention measures. Results shows that undergraduate students of Electronics on the average perform the poorest in year two while the student's average performance then rises slowly to its highest level in year four. These results seem to indicate that undergraduate students of Electronics understudy may on the average need some appropriate remedial actions especially before their penultimate years in the University. Here a Chi-square test statistic was developed and a null hypothesis tested indicated that there exists a statistically significant relationship at the 5% significant level, thereby concluding that the difference between the proportions of undergraduate student of Electronics who on the average have highest and lowest scores through their four years of study in the University are not the same and hence are different. Similar Chi-Square test showed that differential performances in years 1, 2, and 4 are statistically significance at 5% level and may be responsible for the significance of the overall Chi-Square value and hence for the observed differences in performance over the years and hence responsible for the rejection of the initial or overall null hypothesis H_0 of Equation (17). Only the differential performance by students in year 3 is found not to be statistically significant and hence may not be statistically different from 0.25. Given the overall performance, we conclude that about seven undergraduate students of Electronics would therefore seem to need intensive and structured remedial measures to enable them enhance their academic performance in Electronics in the University.

REFERENCES

1. Blanche P, Dartigues JF, Jacqmin-Gadda H. Review and comparison of ROC curve estimators for a time-dependent outcome with marker-dependent censoring. *Biom J*. 2013;55(5):687–704.
2. Cai T, Gerds TA, Zheng Y, Chen J. Robust Prediction of t-Year Survival with Data from Multiple Studies. *Biometrics*. 2011;67(2):436–44.
3. Hung H, Chiang CT. Estimation methods for time-dependent AUC models with survival data. *Can J Stat Revue Can Stat*. 2010;38(1):8–26.
4. Hung H, Chiang CT. Optimal Composite Markers for Time-Dependent Receiver Operating Characteristic Curves with Censored Survival Data. *Scand J Stat*. 2010;37(4):664–79.
5. Kalbfleisch JD, Prentice RL. *The statistical analysis of failure time data*, vol. 360. New Jersey: Wiley; 2011.
6. Lambert J, Chevret S. Summary measure of discrimination in survival models based on cumulative/dynamic time-dependent ROC curves. *Stat Methods In Med Res*. 2014;25(5):2088–102.
7. Potapov S, Adler W, Schmid M: survAUC: Estimators of Prediction Accuracy for Time-to-Event Data. R package version 1.0-5. In.; 2012.
8. Royston P, Parmar MK. The use of restricted mean survival time to estimate the treatment effect in randomized clinical trials when the proportional hazards assumption is in doubt. *Stat Med*. 2011;30(19):2409–21.
9. Saha P, Heagerty PJ. Time-Dependent Predictive Accuracy in the Presence of Competing Risks. *Biometrics*. 2010; 66:999–1011.
11. Saha-Chaudhuri P, Heagerty PJ. Non-parametric estimation of a time-dependent predictive accuracy curve. *Biostatistics*. 2013;14(1):42–59.
12. Shen W, Ning J, Yuan Y. A direct method to evaluate the time-dependent predictive accuracy for biomarkers. *Biometrics*. 2015;71(2):439–49.
13. Song X, Zhou XH, Ma S. Nonparametric receiver operating characteristic-based evaluation for survival outcomes. *Stat Med*. 2012;31(23):2660–75.